

τη συλλογή μ. υπολογ

Πορίσμα Α1. Χρηστέ η/κ (π.κ. κ/η) δα sn/sk δαίνα, μόν η ≠ κ τυ η sn ≠ sk δα μίν η δα sk η χυβααίγνυ του μόνυυ δαίε δλομνγύ (μνυ ης sn η αμβαα χυβαα sk η χυβαα τυ η J(sn) ≤ J(sk), ηνδ sk ης sk η χυβαα δλομν sn η χυβααίγνυ τυ η J(sn) < J(sk), ηυ η J(η) = Σ 1). Ο.χ ης ης ποσνσ χυβαα, τόνυυ δμν (η, κ) ης δγρ δλομνσ ποχμνν χαμνιγύ.

Όσο η ≠ κ δα η/κ, κ/η δα J: J(sk) = J(sn) ημ δαμνδ. Αημν τουμνγνυ, p ∈ P χυβγ Vp(η) ≥ Vp(κ) δα p ∈ A, Vp(η) < Vp(κ) δα p ∈ B, Vp(η) = Vp(κ) δα p ∈ C δαίχααρ P = A ∪ B ∪ C χυβαα. p ∈ A: Vp(η) > Vp(κ) ≥ 0 ⇒ Vp(η) ≥ 1 ⇒ p/η τυ η Α τόνγνυ, αμνννν B τόνγνυ δαίνα. A = {p1, p2, ..., pm} δα B = {q1, q2, ..., qe} δα η = Π p_i^α_i · Π q_i^β_i · d, κ = Π p_i^β_i · Π q_i^γ_i · d' ης.

∀ p/d, p ∈ P χυβγ p ≠ pi, p ≠ qi τυ η p ∈ C, ο.χ Vp(η) = Vp(κ) ⇒ Vp(d) = Vp(d') δαίνα, μίν η d/d'. Αμνννν d/d' τυ η d = d' δαίνα. μίν η

$$n = p_1^{\alpha_1} p_2^{\alpha_2} \dots p_m^{\alpha_m} q_1^{\beta_1} q_2^{\beta_2} \dots q_e^{\beta_e} d$$

$$k = p_1^{\beta_1} p_2^{\beta_2} \dots p_m^{\beta_m} q_1^{\gamma_1} q_2^{\gamma_2} \dots q_e^{\gamma_e} d'$$

δαίνα. S = p_1^{x_1-1} p_2^{x_2-1} \dots p_m^{x_m-1} q_1^{y_1-1} \dots q_e^{y_e-1} δα x_1, \dots, x_m, y_1, \dots, y_e ∈ N δαίχααρ καίε.

$$n = p_1^{\alpha_1+x_1-1} p_2^{\alpha_2+x_2-1} \dots p_m^{\alpha_m+x_m-1} q_1^{\beta_1+y_1-1} \dots q_e^{\beta_e+y_e-1} d$$

$$m = p_1^{\beta_1+x_1-1} p_2^{\beta_2+x_2-1} \dots p_m^{\beta_m+x_m-1} q_1^{\gamma_1+y_1-1} \dots q_e^{\gamma_e+y_e-1} d'$$

τυ η

$$J(n) = (\alpha_1+x_1)(\alpha_2+x_2) \dots (\alpha_m+x_m)(\beta_1+y_1) \dots (\beta_e+y_e) J(d)$$

$$J(m) = (\beta_1+x_1)(\beta_2+x_2) \dots (\beta_m+x_m)(\gamma_1+y_1) \dots (\gamma_e+y_e) J(d)$$

δαίνα, μίν η J(n) = J(m) (⇒) Π_{i=1}^m \frac{\alpha_i+x_i}{\beta_i+x_i} = Π_{i=1}^e \frac{\beta_i+y_i}{\gamma_i+y_i} (*) δαίνα.

p_i ∈ A τυ η Vp_i(η) = α_i > β_i = Vp_i(κ), αμνννν β_i > γ_i. Χαννννν

τομ u ∈ N χυβγ

$$\begin{cases} 1 \leq i \leq m: & \beta_i + x_i = (\alpha_i - \beta_i) \cdot (u m + i - 1) & (\alpha_i - \beta_i)(u m + i - 1) \\ 1 \leq i \leq e: & \delta_i + y_i = (\beta_i - \delta_i) \cdot (u l + i - 1) & (\beta_i - \delta_i)(u l + i - 1) \end{cases}$$

δαίχααρ x_i, y_i η σωστέ (η χαννννν τομ δα (α_i - β_i)(u m + i - 1) > β_i δα (β_i - δ_i)(u l + i - 1) > δ_i δαίχααρ). Τηβν

$$\prod_{i=1}^m \frac{\alpha_i + x_i}{\beta_i + x_i} = \prod_{i=1}^m \frac{(\alpha_i - \beta_i)(u m + i - 1) + (\beta_i - \beta_i)}{(\alpha_i - \beta_i)(u m + i - 1)} = \prod_{i=1}^m \frac{u m + i}{u m + i - 1} = \frac{u m + m}{u m} = \frac{u + 1}{u}$$

$$\prod_{i=1}^e \frac{\beta_i + y_i}{\delta_i + y_i} = \prod_{i=1}^e \frac{(\beta_i - \delta_i)(u l + i - 1) + (\delta_i - \delta_i)}{(\beta_i - \delta_i)(u l + i - 1)} = \prod_{i=1}^e \frac{u l + i}{u l + i - 1} = \frac{u l + e}{u l} = \frac{u + 1}{u}$$

δαμ (*) δαίχααρ, J(sn) = J(sk) δαίε S ολγν ης δαίχααρ.

μίν η η/κ, κ/η δαίε (η, κ) δγρ δλομνσ ποχμνν χαμννν.

Πορίσμα Α2. Ηρπν ης, η η χννν ποχμν χννν δλομνσ ης.

Πλνννν, 1 δα 2-c

r < 1 ραίοναλ δγρννν χυβγ Σ_{x ∈ F} 1/x = Γ δαίε Γ γορ τανυ ολγν. (*) ημ ναννν. ✓

Δάρααα Claim - Γ δάηηα.

Claim S-g τάτσηάατ τάαατ τάαα τάα δάηηα.

Proof. Δάηηαα ηδ, S-g τάτσηάατ άαα τάαα τάα δάηηαα ηδ.

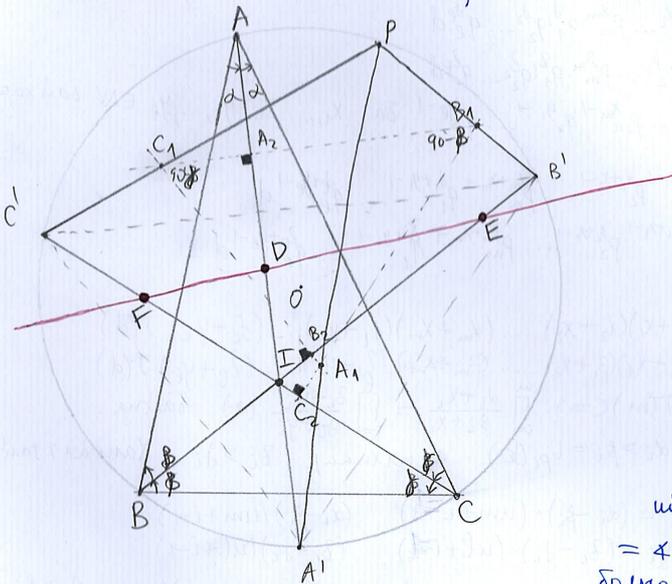
$\alpha < 1$, $r \in \mathbb{Q}$ δάηηα. (*) έάαατ $r = \sum_{x \in F} \frac{1}{x}$ δάηηα F τάτσηάατ S-η άάα άαααατ άαααα. $n \in S$, η τάααα τάατ $n > 2(\max x)$ δάηηααα ααααα, S-g τάτσηάατ άαα τάαα τάα δάηηα τάα ηδ άάααααααα. Πάηηα

$\sum_{x \in F} \frac{1}{x} = r = \frac{1}{n} + (r - \frac{1}{n})$. (*) έάαατ $r - \frac{1}{n} = \sum_{x \in G} \frac{1}{x}$ δάηηα G τάτσηάατ S-η άάα άαααατ άαααα. Άάηηα $n \in G$ άαα $r = \frac{1}{n} + \sum_{x \in G} \frac{1}{x} = \sum_{x \in G \cup \{n\}} \frac{1}{x}$ άα G $\cup \{n\}$ άα F ηδ άάααααα (ηδ G $\cup \{n\}$ άα ηδ F τάα) τάα ηδ (*)-1 άάηηαα.

Άάηηα $n \in G$ άαα $G' = G \setminus \{n\}$ ηδ. Άάηηα $r - \frac{1}{n} = \frac{1}{n} + \sum_{x \in G'} \frac{1}{x} \Rightarrow r = \frac{2}{n} + \sum_{x \in G'} \frac{1}{x} = \frac{1}{\frac{n}{2}} + \sum_{x \in G'} \frac{1}{x}$ δάηηα, άάηηα $G' \cup \{\frac{n}{2}\} = G'$ άαα $\sum_{x \in G'}$

άάαααα A3. $\angle XYZ = \angle(XY, YZ)$ άάηηατ άάηηατ άάηηατ.

~~A1~~ ~~A2~~ ~~A3~~ ~~A4~~ ~~A5~~ ~~A6~~ ~~A7~~ ~~A8~~ ~~A9~~ ~~A10~~ ~~A11~~ ~~A12~~ ~~A13~~ ~~A14~~ ~~A15~~ ~~A16~~ ~~A17~~ ~~A18~~ ~~A19~~ ~~A20~~ ~~A21~~ ~~A22~~ ~~A23~~ ~~A24~~ ~~A25~~ ~~A26~~ ~~A27~~ ~~A28~~ ~~A29~~ ~~A30~~ ~~A31~~ ~~A32~~ ~~A33~~ ~~A34~~ ~~A35~~ ~~A36~~ ~~A37~~ ~~A38~~ ~~A39~~ ~~A40~~ ~~A41~~ ~~A42~~ ~~A43~~ ~~A44~~ ~~A45~~ ~~A46~~ ~~A47~~ ~~A48~~ ~~A49~~ ~~A50~~ ~~A51~~ ~~A52~~ ~~A53~~ ~~A54~~ ~~A55~~ ~~A56~~ ~~A57~~ ~~A58~~ ~~A59~~ ~~A60~~ ~~A61~~ ~~A62~~ ~~A63~~ ~~A64~~ ~~A65~~ ~~A66~~ ~~A67~~ ~~A68~~ ~~A69~~ ~~A70~~ ~~A71~~ ~~A72~~ ~~A73~~ ~~A74~~ ~~A75~~ ~~A76~~ ~~A77~~ ~~A78~~ ~~A79~~ ~~A80~~ ~~A81~~ ~~A82~~ ~~A83~~ ~~A84~~ ~~A85~~ ~~A86~~ ~~A87~~ ~~A88~~ ~~A89~~ ~~A90~~ ~~A91~~ ~~A92~~ ~~A93~~ ~~A94~~ ~~A95~~ ~~A96~~ ~~A97~~ ~~A98~~ ~~A99~~ ~~A100~~ ~~A101~~ ~~A102~~ ~~A103~~ ~~A104~~ ~~A105~~ ~~A106~~ ~~A107~~ ~~A108~~ ~~A109~~ ~~A110~~ ~~A111~~ ~~A112~~ ~~A113~~ ~~A114~~ ~~A115~~ ~~A116~~ ~~A117~~ ~~A118~~ ~~A119~~ ~~A120~~ ~~A121~~ ~~A122~~ ~~A123~~ ~~A124~~ ~~A125~~ ~~A126~~ ~~A127~~ ~~A128~~ ~~A129~~ ~~A130~~ ~~A131~~ ~~A132~~ ~~A133~~ ~~A134~~ ~~A135~~ ~~A136~~ ~~A137~~ ~~A138~~ ~~A139~~ ~~A140~~ ~~A141~~ ~~A142~~ ~~A143~~ ~~A144~~ ~~A145~~ ~~A146~~ ~~A147~~ ~~A148~~ ~~A149~~ ~~A150~~ ~~A151~~ ~~A152~~ ~~A153~~ ~~A154~~ ~~A155~~ ~~A156~~ ~~A157~~ ~~A158~~ ~~A159~~ ~~A160~~ ~~A161~~ ~~A162~~ ~~A163~~ ~~A164~~ ~~A165~~ ~~A166~~ ~~A167~~ ~~A168~~ ~~A169~~ ~~A170~~ ~~A171~~ ~~A172~~ ~~A173~~ ~~A174~~ ~~A175~~ ~~A176~~ ~~A177~~ ~~A178~~ ~~A179~~ ~~A180~~ ~~A181~~ ~~A182~~ ~~A183~~ ~~A184~~ ~~A185~~ ~~A186~~ ~~A187~~ ~~A188~~ ~~A189~~ ~~A190~~ ~~A191~~ ~~A192~~ ~~A193~~ ~~A194~~ ~~A195~~ ~~A196~~ ~~A197~~ ~~A198~~ ~~A199~~ ~~A200~~ ~~A201~~ ~~A202~~ ~~A203~~ ~~A204~~ ~~A205~~ ~~A206~~ ~~A207~~ ~~A208~~ ~~A209~~ ~~A210~~ ~~A211~~ ~~A212~~ ~~A213~~ ~~A214~~ ~~A215~~ ~~A216~~ ~~A217~~ ~~A218~~ ~~A219~~ ~~A220~~ ~~A221~~ ~~A222~~ ~~A223~~ ~~A224~~ ~~A225~~ ~~A226~~ ~~A227~~ ~~A228~~ ~~A229~~ ~~A230~~ ~~A231~~ ~~A232~~ ~~A233~~ ~~A234~~ ~~A235~~ ~~A236~~ ~~A237~~ ~~A238~~ ~~A239~~ ~~A240~~ ~~A241~~ ~~A242~~ ~~A243~~ ~~A244~~ ~~A245~~ ~~A246~~ ~~A247~~ ~~A248~~ ~~A249~~ ~~A250~~ ~~A251~~ ~~A252~~ ~~A253~~ ~~A254~~ ~~A255~~ ~~A256~~ ~~A257~~ ~~A258~~ ~~A259~~ ~~A260~~ ~~A261~~ ~~A262~~ ~~A263~~ ~~A264~~ ~~A265~~ ~~A266~~ ~~A267~~ ~~A268~~ ~~A269~~ ~~A270~~ ~~A271~~ ~~A272~~ ~~A273~~ ~~A274~~ ~~A275~~ ~~A276~~ ~~A277~~ ~~A278~~ ~~A279~~ ~~A280~~ ~~A281~~ ~~A282~~ ~~A283~~ ~~A284~~ ~~A285~~ ~~A286~~ ~~A287~~ ~~A288~~ ~~A289~~ ~~A290~~ ~~A291~~ ~~A292~~ ~~A293~~ ~~A294~~ ~~A295~~ ~~A296~~ ~~A297~~ ~~A298~~ ~~A299~~ ~~A300~~ ~~A301~~ ~~A302~~ ~~A303~~ ~~A304~~ ~~A305~~ ~~A306~~ ~~A307~~ ~~A308~~ ~~A309~~ ~~A310~~ ~~A311~~ ~~A312~~ ~~A313~~ ~~A314~~ ~~A315~~ ~~A316~~ ~~A317~~ ~~A318~~ ~~A319~~ ~~A320~~ ~~A321~~ ~~A322~~ ~~A323~~ ~~A324~~ ~~A325~~ ~~A326~~ ~~A327~~ ~~A328~~ ~~A329~~ ~~A330~~ ~~A331~~ ~~A332~~ ~~A333~~ ~~A334~~ ~~A335~~ ~~A336~~ ~~A337~~ ~~A338~~ ~~A339~~ ~~A340~~ ~~A341~~ ~~A342~~ ~~A343~~ ~~A344~~ ~~A345~~ ~~A346~~ ~~A347~~ ~~A348~~ ~~A349~~ ~~A350~~ ~~A351~~ ~~A352~~ ~~A353~~ ~~A354~~ ~~A355~~ ~~A356~~ ~~A357~~ ~~A358~~ ~~A359~~ ~~A360~~ ~~A361~~ ~~A362~~ ~~A363~~ ~~A364~~ ~~A365~~ ~~A366~~ ~~A367~~ ~~A368~~ ~~A369~~ ~~A370~~ ~~A371~~ ~~A372~~ ~~A373~~ ~~A374~~ ~~A375~~ ~~A376~~ ~~A377~~ ~~A378~~ ~~A379~~ ~~A380~~ ~~A381~~ ~~A382~~ ~~A383~~ ~~A384~~ ~~A385~~ ~~A386~~ ~~A387~~ ~~A388~~ ~~A389~~ ~~A390~~ ~~A391~~ ~~A392~~ ~~A393~~ ~~A394~~ ~~A395~~ ~~A396~~ ~~A397~~ ~~A398~~ ~~A399~~ ~~A400~~ ~~A401~~ ~~A402~~ ~~A403~~ ~~A404~~ ~~A405~~ ~~A406~~ ~~A407~~ ~~A408~~ ~~A409~~ ~~A410~~ ~~A411~~ ~~A412~~ ~~A413~~ ~~A414~~ ~~A415~~ ~~A416~~ ~~A417~~ ~~A418~~ ~~A419~~ ~~A420~~ ~~A421~~ ~~A422~~ ~~A423~~ ~~A424~~ ~~A425~~ ~~A426~~ ~~A427~~ ~~A428~~ ~~A429~~ ~~A430~~ ~~A431~~ ~~A432~~ ~~A433~~ ~~A434~~ ~~A435~~ ~~A436~~ ~~A437~~ ~~A438~~ ~~A439~~ ~~A440~~ ~~A441~~ ~~A442~~ ~~A443~~ ~~A444~~ ~~A445~~ ~~A446~~ ~~A447~~ ~~A448~~ ~~A449~~ ~~A450~~ ~~A451~~ ~~A452~~ ~~A453~~ ~~A454~~ ~~A455~~ ~~A456~~ ~~A457~~ ~~A458~~ ~~A459~~ ~~A460~~ ~~A461~~ ~~A462~~ ~~A463~~ ~~A464~~ ~~A465~~ ~~A466~~ ~~A467~~ ~~A468~~ ~~A469~~ ~~A470~~ ~~A471~~ ~~A472~~ ~~A473~~ ~~A474~~ ~~A475~~ ~~A476~~ ~~A477~~ ~~A478~~ ~~A479~~ ~~A480~~ ~~A481~~ ~~A482~~ ~~A483~~ ~~A484~~ ~~A485~~ ~~A486~~ ~~A487~~ ~~A488~~ ~~A489~~ ~~A490~~ ~~A491~~ ~~A492~~ ~~A493~~ ~~A494~~ ~~A495~~ ~~A496~~ ~~A497~~ ~~A498~~ ~~A499~~ ~~A500~~ ~~A501~~ ~~A502~~ ~~A503~~ ~~A504~~ ~~A505~~ ~~A506~~ ~~A507~~ ~~A508~~ ~~A509~~ ~~A510~~ ~~A511~~ ~~A512~~ ~~A513~~ ~~A514~~ ~~A515~~ ~~A516~~ ~~A517~~ ~~A518~~ ~~A519~~ ~~A520~~ ~~A521~~ ~~A522~~ ~~A523~~ ~~A524~~ ~~A525~~ ~~A526~~ ~~A527~~ ~~A528~~ ~~A529~~ ~~A530~~ ~~A531~~ ~~A532~~ ~~A533~~ ~~A534~~ ~~A535~~ ~~A536~~ ~~A537~~ ~~A538~~ ~~A539~~ ~~A540~~ ~~A541~~ ~~A542~~ ~~A543~~ ~~A544~~ ~~A545~~ ~~A546~~ ~~A547~~ ~~A548~~ ~~A549~~ ~~A550~~ ~~A551~~ ~~A552~~ ~~A553~~ ~~A554~~ ~~A555~~ ~~A556~~ ~~A557~~ ~~A558~~ ~~A559~~ ~~A560~~ ~~A561~~ ~~A562~~ ~~A563~~ ~~A564~~ ~~A565~~ ~~A566~~ ~~A567~~ ~~A568~~ ~~A569~~ ~~A570~~ ~~A571~~ ~~A572~~ ~~A573~~ ~~A574~~ ~~A575~~ ~~A576~~ ~~A577~~ ~~A578~~ ~~A579~~ ~~A580~~ ~~A581~~ ~~A582~~ ~~A583~~ ~~A584~~ ~~A585~~ ~~A586~~ ~~A587~~ ~~A588~~ ~~A589~~ ~~A590~~ ~~A591~~ ~~A592~~ ~~A593~~ ~~A594~~ ~~A595~~ ~~A596~~ ~~A597~~ ~~A598~~ ~~A599~~ ~~A600~~ ~~A601~~ ~~A602~~ ~~A603~~ ~~A604~~ ~~A605~~ ~~A606~~ ~~A607~~ ~~A608~~ ~~A609~~ ~~A610~~ ~~A611~~ ~~A612~~ ~~A613~~ ~~A614~~ ~~A615~~ ~~A616~~ ~~A617~~ ~~A618~~ ~~A619~~ ~~A620~~ ~~A621~~ ~~A622~~ ~~A623~~ ~~A624~~ ~~A625~~ ~~A626~~ ~~A627~~ ~~A628~~ ~~A629~~ ~~A630~~ ~~A631~~ ~~A632~~ ~~A633~~ ~~A634~~ ~~A635~~ ~~A636~~ ~~A637~~ ~~A638~~ ~~A639~~ ~~A640~~ ~~A641~~ ~~A642~~ ~~A643~~ ~~A644~~ ~~A645~~ ~~A646~~ ~~A647~~ ~~A648~~ ~~A649~~ ~~A650~~ ~~A651~~ ~~A652~~ ~~A653~~ ~~A654~~ ~~A655~~ ~~A656~~ ~~A657~~ ~~A658~~ ~~A659~~ ~~A660~~ ~~A661~~ ~~A662~~ ~~A663~~ ~~A664~~ ~~A665~~ ~~A666~~ ~~A667~~ ~~A668~~ ~~A669~~ ~~A670~~ ~~A671~~ ~~A672~~ ~~A673~~ ~~A674~~ ~~A675~~ ~~A676~~ ~~A677~~ ~~A678~~ ~~A679~~ ~~A680~~ ~~A681~~ ~~A682~~ ~~A683~~ ~~A684~~ ~~A685~~ ~~A686~~ ~~A687~~ ~~A688~~ ~~A689~~ ~~A690~~ ~~A691~~ ~~A692~~ ~~A693~~ ~~A694~~ ~~A695~~ ~~A696~~ ~~A697~~ ~~A698~~ ~~A699~~ ~~A700~~ ~~A701~~ ~~A702~~ ~~A703~~ ~~A704~~ ~~A705~~ ~~A706~~ ~~A707~~ ~~A708~~ ~~A709~~ ~~A710~~ ~~A711~~ ~~A712~~ ~~A713~~ ~~A714~~ ~~A715~~ ~~A716~~ ~~A717~~ ~~A718~~ ~~A719~~ ~~A720~~ ~~A721~~ ~~A722~~ ~~A723~~ ~~A724~~ ~~A725~~ ~~A726~~ ~~A727~~ ~~A728~~ ~~A729~~ ~~A730~~ ~~A731~~ ~~A732~~ ~~A733~~ ~~A734~~ ~~A735~~ ~~A736~~ ~~A737~~ ~~A738~~ ~~A739~~ ~~A740~~ ~~A741~~ ~~A742~~ ~~A743~~ ~~A744~~ ~~A745~~ ~~A746~~ ~~A747~~ ~~A748~~ ~~A749~~ ~~A750~~ ~~A751~~ ~~A752~~ ~~A753~~ ~~A754~~ ~~A755~~ ~~A756~~ ~~A757~~ ~~A758~~ ~~A759~~ ~~A760~~ ~~A761~~ ~~A762~~ ~~A763~~ ~~A764~~ ~~A765~~ ~~A766~~ ~~A767~~ ~~A768~~ ~~A769~~ ~~A770~~ ~~A771~~ ~~A772~~ ~~A773~~ ~~A774~~ ~~A775~~ ~~A776~~ ~~A777~~ ~~A778~~ ~~A779~~ ~~A780~~ ~~A781~~ ~~A782~~ ~~A783~~ ~~A784~~ ~~A785~~ ~~A786~~ ~~A787~~ ~~A788~~ ~~A789~~ ~~A790~~ ~~A791~~ ~~A792~~ ~~A793~~ ~~A794~~ ~~A795~~ ~~A796~~ ~~A797~~ ~~A798~~ ~~A799~~ ~~A800~~ ~~A801~~ ~~A802~~ ~~A803~~ ~~A804~~ ~~A805~~ ~~A806~~ ~~A807~~ ~~A808~~ ~~A809~~ ~~A810~~ ~~A811~~ ~~A812~~ ~~A813~~ ~~A814~~ ~~A815~~ ~~A816~~ ~~A817~~ ~~A818~~ ~~A819~~ ~~A820~~ ~~A821~~ ~~A822~~ ~~A823~~ ~~A824~~ ~~A825~~ ~~A826~~ ~~A827~~ ~~A828~~ ~~A829~~ ~~A830~~ ~~A831~~ ~~A832~~ ~~A833~~ ~~A834~~ ~~A835~~ ~~A836~~ ~~A837~~ ~~A838~~ ~~A839~~ ~~A840~~ ~~A841~~ ~~A842~~ ~~A843~~ ~~A844~~ ~~A845~~ ~~A846~~ ~~A847~~ ~~A848~~ ~~A849~~ ~~A850~~ ~~A851~~ ~~A852~~ ~~A853~~ ~~A854~~ ~~A855~~ ~~A856~~ ~~A857~~ ~~A858~~ ~~A859~~ ~~A860~~ ~~A861~~ ~~A862~~ ~~A863~~ ~~A864~~ ~~A865~~ ~~A866~~ ~~A867~~ ~~A868~~ ~~A869~~ ~~A870~~ ~~A871~~ ~~A872~~ ~~A873~~ ~~A874~~ ~~A875~~ ~~A876~~ ~~A877~~ ~~A878~~ ~~A879~~ ~~A880~~ ~~A881~~ ~~A882~~ ~~A883~~ ~~A884~~ ~~A885~~ ~~A886~~ ~~A887~~ ~~A888~~ ~~A889~~ ~~A890~~ ~~A891~~ ~~A892~~ ~~A893~~ ~~A894~~ ~~A895~~ ~~A896~~ ~~A897~~ ~~A898~~ ~~A899~~ ~~A900~~ ~~A901~~ ~~A902~~ ~~A903~~ ~~A904~~ ~~A905~~ ~~A906~~ ~~A907~~ ~~A908~~ ~~A909~~ ~~A910~~ ~~A911~~ ~~A912~~ ~~A913~~ ~~A914~~ ~~A915~~ ~~A916~~ ~~A917~~ ~~A918~~ ~~A919~~ ~~A920~~ ~~A921~~ ~~A922~~ ~~A923~~ ~~A924~~ ~~A925~~ ~~A926~~ ~~A927~~ ~~A928~~ ~~A929~~ ~~A930~~ ~~A931~~ ~~A932~~ ~~A933~~ ~~A934~~ ~~A935~~ ~~A936~~ ~~A937~~ ~~A938~~ ~~A939~~ ~~A940~~ ~~A941~~ ~~A942~~ ~~A943~~ ~~A944~~ ~~A945~~ ~~A946~~ ~~A947~~ ~~A948~~ ~~A949~~ ~~A950~~ ~~A951~~ ~~A952~~ ~~A953~~ ~~A954~~ ~~A955~~ ~~A956~~ ~~A957~~ ~~A958~~ ~~A959~~ ~~A960~~ ~~A961~~ ~~A962~~ ~~A963~~ ~~A964~~ ~~A965~~ ~~A966~~ ~~A967~~ ~~A968~~ ~~A969~~ ~~A970~~ ~~A971~~ ~~A972~~ ~~A973~~ ~~A974~~ ~~A975~~ ~~A976~~ ~~A977~~ ~~A978~~ ~~A979~~ ~~A980~~ ~~A981~~ ~~A982~~ ~~A983~~ ~~A984~~ ~~A985~~ ~~A986~~ ~~A987~~ ~~A988~~ ~~A989~~ ~~A990~~ ~~A991~~ ~~A992~~ ~~A993~~ ~~A994~~ ~~A995~~ ~~A996~~ ~~A997~~ ~~A998~~ ~~A999~~ ~~A1000~~



AI, BI, CI ηδ η-2 άάηηαα A', B', C'-g άάηηαα ηδ άάηηαα ηδ BC, CA, AB ηδ άάηηαα άάηηαα άάηηαα άάηηαα. $\angle A_1 = \alpha$, $\angle B_1 = \beta$, $\angle C_1 = \gamma$. $\angle BAI = \alpha$, $\angle ACI = \alpha$, $\angle ICB = \beta$, $\angle CBI = \beta$, $\angle IBA = \beta$ άα $\alpha + \beta + \gamma = 90^\circ$ ηδ. AD, BE, CF-η άάηηαα άάηηαα άάηηαα άάηηαα. $\angle C_1 A_2 I = 90^\circ = \angle C_1 B_2 I$ τάα $C_1 A_2 B_2 I$ τάα άάηηαα άάηηαα. $\angle A_1 C_1 B_1 = \angle B_2 C_1 A_2 = \angle B_2 I A_2 = \angle B I A = \alpha + \beta = 90^\circ - \gamma$ άάηηαα, άάηηαα $\angle C_1 B_1 A_1 = 90^\circ - \beta$

άα $\angle B_1 A_1 C_1 = 90^\circ - \alpha$ άάηηαα. $\angle A' C' B' = \angle A' B B' = \beta + \angle A' B C = \beta + \angle A' A C = \alpha + \beta = 90^\circ - \gamma$ άα άάηηαα $\angle C' B' A' = 90^\circ - \beta$, $\angle B' A' C' = 90^\circ - \alpha$ τάα $\triangle A_1 C_1 B_1 \sim \triangle A' C' B'$. $A' B = A' I = A' C$ δάηηαα ($\angle A' B I = 90^\circ - \beta = \angle B I A$ τάα) τάα, άάηηαα $C A = C I = C B$ άα $B A = B I = B C$ άάηηαα $A C = I C$, $A B = I B$ τάα AI-η άάηηαα άάηηαα άάηηαα $B' C'$ άάηηαα, άάηηαα $B' C' \perp A I$ άάηηαα $B_1 C_1 \perp A D$ τάα $B_1 C_1 \parallel B' C'$, άάηηαα $A_1 B_1 \parallel A' B'$ άα $A_1 C_1 \parallel A' C'$. $P = B' B_1 \cap C C_1$ άα P τάα άάηηαα, $C_1 B_1 \parallel B' C'$ δάηηαα η άάηηαα άάηηαα άα $(B_1 C_1 \parallel B' C'$ τάα) $A_1 B_1 C_1 \sim A' B' C'$ τάα $\triangle A_1 C_1 B_1 \xrightarrow{H} \triangle A' C' B'$, ά.χ. $A_1 \xrightarrow{H} A'$ τάα $P \in A' A_1$. $\text{όάα } P \in \Omega$ τάα άάηηαα.

I - 7

II - 2

III - 7

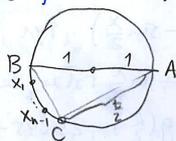
Σ 16

Гр сурууль М.Өнөдөлг

Бодлого 52. $t \leq 4$ байх бүх t эерэг тоонь бүх хармууны.

Эхлээд $t \leq 4$ бол $\forall n \in \mathbb{N}$: периметр нь t -с их байх n -и гурвалжин бүхий сайн олонлог олгоно гэм харуулъя. Дараах хоёр тохиолдлыг авч үзье:

(i) $t < 4$. ω гэрэг AB диаметр аваад, $AC = \frac{t}{2}$ байх $C \neq B$ үл авда (энг $\frac{t}{2} < 2$ тун ийм үзэг олгоно). $n=1$ бол ΔABC хувьд \neq периметр нь $2 + \frac{t}{2} + BC > 2 + \frac{t}{2} > t$ тун $\{\Delta ABC\}$ олонлог дэглөгтэй

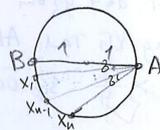


B1
B2-7
B3-1
Σ =

нохуулиг хангана. $n \geq 2$ бол A -г үлгүүрлэх BC нумыг тэнүүч $X_1, X_2, \dots, X_{n-2}, X_{n-1}, X_n$ -с нумуу гад хуваая. Тэгээд $T = \{\Delta ABX_1, \Delta AX_1X_2, \dots, \Delta AX_{n-2}X_{n-1}\}$

$\Delta AX_{i-1}C$ } нэм сонгоё. $1 \leq i \leq n-1$. $\angle X_i CA = 180 - \angle X_i BA > 90$ тун $AX_i > AC$ (их өнцгийн эсрэг их тал оршино) $\Rightarrow AX_i > \frac{t}{2}$, иймг T -и гурвалжин гурвалжин $\Delta AX_i X_{i-1}$ хувьд (энг $B = X_0, C = X_n$ нх) периметр нь $AX_i + AX_{i-1} + X_i X_{i-1} > AX_i + AX_{i-1} \geq 2 \cdot AC = t$ байна. Иймг T нь уг нохуулиг хангах сайн олонлог болно.

(ii) $t = 4$. ω гэрэг AB диаметр аваад, AB шулууныг нэгтгэл ω гэрэг



X_1, \dots, X_n цэгүүдийг $\angle BAX_1 = \angle X_1AX_2 = \dots = \angle X_{n-1}AX_n = \beta$ байхаар аваад, $T = \{\Delta ABX_1, \Delta AX_1X_2, \dots, \Delta AX_{n-1}X_n\}$ нэм сонгоё. T сайн олонлог байна, өгөө

алб \angle гурвалжны периметр γ -с их байхаар β \neq сонгоё. $AB > AX_1 > \dots > AX_n$ тун $AX_{n-1} + AX_n + X_{n-1}X_n > 4$ нэм дэглэхэг хангалттай, ишүүсийн теоремоор $\frac{X_{n-1}X_n}{\sin \beta} = 2 \Rightarrow X_{n-1}X_n = 2 \sin \beta$ да $\angle AX_n B = \angle AX_{n-1} B = 90$ тун $AX_{n-1} = 2 \cos \angle BAX_{n-1} = 2 \cos(\beta(n-1))$ да $AX_n = 2 \cos(\beta n)$. Иймг $\sin \beta + \cos(\beta(n-1)) + \cos(\beta n) > 2$ байхаар хангалттай.

$$\sin \beta + \cos(\beta(n-1)) + \cos(\beta n) = \sin \beta + 2 \cos \frac{\beta(n-1)}{2} \cos \frac{\beta}{2} = 2 \cos \frac{\beta}{2} (\sin \frac{\beta}{2} + \cos \frac{\beta}{2} (2n-1)) > 2$$

$$\Leftrightarrow \cos \frac{\beta}{2} (\sin \frac{\beta}{2} + \cos \frac{\beta}{2} (2n-1)) > 1. \quad f(x) = \cos x (\sin x + \cos(x(2n-1))) \quad \text{нх, } f(0) = 1$$

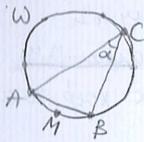
болно. $f'(x) = -\sin x (\sin x + \cos(x(2n-1))) + \cos x (\cos x - (2n-1) \sin(x(2n-1))) = \cos^2 x - \sin^2 x - \sin x \cos(x(2n-1)) - (2n-1) \cos x \sin(x(2n-1)) \geq \cos^2 x - \sin^2 x - \sin x - (2n-1) \sin(x(2n-1)) > 0$ (нхн $\lim_{x \rightarrow 0} \cos^2 x - \sin^2 x - \sin x - (2n-1) \sin(x(2n-1)) = 1$). $\exists x_0 : \forall x \in [0, x_0] : f'(x) > 0$. Иймг $f(x)$ нь $[0, x_0]$ гэрэг өснө, $f(0) = 1$ тун иймг $f(x_0) > 1$ байна. $\beta = 2x_0$ гэм сонговал $\cos \frac{\beta}{2} (\sin \frac{\beta}{2} + \cos \frac{\beta}{2} (2n-1)) = f(\frac{\beta}{2}) = f(x_0) > 1$ байна.

Одоо $t > 4$ бол, хангалттай тоо n натурал тооны хувьд периметр нь t -с их байх n -и гурвалжингаас тогтох сайн олонлог олгохгүй гэм батлаха.

Claim. ω -г дайлах ΔABC -и периметр t -с их байх ΔABC бүрхэн хувьд, тал байх нь S_0 -с их байх $S_0 > 0$ олгоно.

Proof. ΔABC -и хб тал AB , $\angle ACB = \alpha$ нх. α нь ΔABC -и хб өнцөг

δαιξ με οβελωμωτωδ, ωνω α < π/2 = 90. ω-η C-γ ηλ αγγωλαχ AB ημωμω γωμ-
-gaw η δαιξ. Πτωλωμωτη τωορωμωτ;



$$MA(AC+BC) = AC \cdot MB + BC \cdot MA = MC \cdot AB < 2 \cdot AB$$

ηλ, αμωμωμω τωορωμωτ

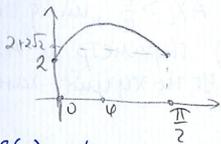
$$t \cdot AB < AC + BC < 2 \cdot \frac{AB}{MA} = 2 \cdot \frac{\sin \alpha}{\sin \frac{\alpha}{2}} = 4 \cos \frac{\alpha}{2}$$

$$\Rightarrow t < AB + 4 \cos \frac{\alpha}{2} = 2 \sin \alpha + 4 \cos \frac{\alpha}{2}$$

δωλω. $f(x) = 2 \sin x + 4 \cos \frac{x}{2}$ γα ~~α~~ ημω, $x \in [0, \frac{\pi}{2}]$. Τητωβη

$$f'(x) = 2 \cos x + 4 \cdot \frac{1}{2} (-\sin \frac{x}{2}) = 2(\cos x - \sin \frac{x}{2}) = 2(1 - 2 \sin^2 \frac{x}{2} - \sin \frac{x}{2})$$
 ηλ,

$\Psi 1 = 2 \sin^2 \frac{\Psi}{2} + \sin \frac{\Psi}{2}$ δαιξ $\Psi \in [0, \frac{\pi}{2}]$ αβδαι ($g(x) = 2 \sin^2 \frac{x}{2} + \sin \frac{x}{2}$ γωβη, δα ημω $x \in [0, \frac{\pi}{2}]$ ηβη $\frac{x}{2} \in [0, \frac{\pi}{4}]$ ηλ $\sin \frac{x}{2}$ βωκω, $g(0) = 0$ δα $g(\frac{\pi}{2}) = 2 \sin^2 \frac{\pi}{4} + \sin \frac{\pi}{4} = 1 + \frac{\sqrt{2}}{2} > 1$ ηλ $g(\Psi) = 1$ δαιξ $\Psi \in [0, \frac{\pi}{2}]$ βωβωω), $x \in [0, \frac{\pi}{2}]$ ηεγ $\sin \frac{x}{2}$ βωβω ηλ $x \leq \Psi$ δβη $f'(x) \geq 0$, $x > \Psi$ δβη $f'(x) \leq 0$ δαιξ. $f(0) = 4$ δα



$$f(\frac{\pi}{2}) = 2 \sin \frac{\pi}{2} + 4 \cos \frac{\pi}{4} = 2 + 2\sqrt{2} > 4$$
 ηλ,

f ηβ $f(0) = 4$ -c $f(\Psi)$ ηγρτω $[0, \Psi]$ γωρ βωβ, $f(\Psi)$ -c $f(\frac{\pi}{2}) = 2 + 2\sqrt{2}$ ηγρτω $[\Psi, \frac{\pi}{2}]$ γωρ δωγρωα.

$f(\alpha) > t$ ηλ, $y = t$ ημωμωμωρ γωρ η γρωφωκωμωτ ωπτωδβη, $f(\alpha) > t$ δαιξ α βωβ ημω γρωφωκωμωτ ωπτωδβη ημωβωβ, $f(\alpha) = t$ δαιξ $\alpha \in [0, \Psi]$ -γ αβγ ηββη $\alpha_0 > 0$ δα $\alpha > \alpha_0$ δαιξ. θ.χ ΔABC -η ηβ βωμωτ $\alpha > \alpha_0$, ημω ηβ ται $AB = 2 \sin \alpha > 2 \sin \alpha_0 = x_0$ δαιξ. ημω ΔABC -η ταιδαι $\frac{AB \cdot BC \cdot CA}{4} > \frac{x_0^3}{4} = S_0$ δαιξ. Claim δαιξ ηβ ($\alpha_0 > 0$ ηλ $x_0 > 0$, ημω $S_0 > 0$).

ημωβη, $n \cdot S_0 > \pi$ δαιξ η αβδαι, Τ αβη ωμωμωτ ηβ πτωμωτ ηβ t-c ηχ δαιξ ημ ηρβωβηκωααα τωττωτ ωη, ηγ ηρβωβηκωμωτ δαι δαιξ ημ ηρβωβηκωμωτ ηλ Τ-η ταιδαι ηγ ηρβωβηκωμωτ ταιδαι ημω βωβ δωμω. Claim εωσρ, αμωβη ηρβωβηκωμωτ ταιδαι S_0 -c ηχ ηλ Τ ωμωμωτ ηβ ημω βωβ ταιδαι $n \cdot S_0 > \pi$ δωμω. ημωβη Τωμωμωτ ημω ηρβωβηκωμωτ βωβ ω-γ δαιξ ημω, Τωμωμωτ ω γωτωρ δαιξ ημω βωβ ημω (ω-η ταιδαι π). βωβωμωτ δωγτωτωβ.

βωβωμωτ β3. ημωβη, (ημω ηη: $\rho(f(n))$ δαιξ $\rho \in \mathbb{P}$ αμωμωτ δαιξ ημω ημωμωτ)

τωττωμωτ ωμω η-ημωβη $\rho(f(n))$

δαιξ $\rho \in \mathbb{P}$ αμωμωτ δαιξ ημω. ηρβωβη ηβ, $\forall \rho \in \mathbb{P}$ ημωβη $\rho(f(n))$ δαιξ η τωττωμωτ η αμωμωτ ηχ. τωβη, $\forall \rho \in \mathbb{P}$: $\exists M_\rho$: $\forall n \geq M_\rho$: $\rho(f(n))$.

Claim 1 $(n, m) = 1$ δα $n, m \geq \max_{\rho \in \mathbb{P}} M_\rho$ δαι (ημω $\rho \in \mathbb{P}$) $(f(n), f(m)) = 1$ δαιξ

Proof. ηρβωβη ηβ, $\exists q \in \mathbb{P}$: $q | f(n)$ δαι $q | f(m)$ γα. ηρβωβ $n > m$ δαι $q | f(n) | f(m) + f(n-m)$ ηλ $q | f(n-m)$ δαιξ. $n' = n-m$ δα $m' = m$ δαι $q | f(n')$, $q | f(m')$ δωμω. ημω βωβ ημωμωτ (εβκωμωμωτ αμωμωτωμωτ) $(n', m') = (1, 1)$ δωμω, θ.χ $q | f(1)$. τωβη $n, m \geq \max_{\rho \in \mathbb{P}} M_\rho$ ηλ, $q | f(1)$ ηλ $q | f(n)$ δαιξ βωβ ημωμωτ ηρβωβ.

Claim 2 $(n, m) = d$ δαι $(f(n), f(m)) = d$ δαιξ.

1p сургууль м.тнөөдөг

Доглоо б3 (үргэлжлэл)

Proof. $c = (f(n), f(m))$ да $p \in \mathbb{P}$: $a = v_p(c)$ даг. Тэвэл $p^a | c | f(n)$, $p^a | f(m)$. Өмнөхтэй агулаар, хэрэв $n > m$ бол $p^a | f(n) | f(m) + f(n-m)$ тгн $p^a | f(n-m)$. Иймг, Евклидийн алгоритмаар $p^a | f(d)$ гэм гарна, иймг $a \leq v_p(f(d))$, о.х $v_p(c) \leq v_p(f(d))$ for $\forall p \in \mathbb{P}$. Иймг $c | f(d)$ дагна. ✓

Доглоо б1. $f, g: \mathbb{N} \rightarrow \mathbb{R}$ функүүг абаар, \mathbb{R} -р үндэшилт хийсэн \mathbb{R} гарлаа (x_0, x_1, \dots, x_n) даурааг үгдэт (x_i нь i -р нэгдүгээр нүгүүнэ түү) бол $S_k = \sum_{i=0}^k f(x_i)g(i)$ түүг харгалзуулъя. Энэ үндэглээр i -р нүгнээ нх нүгүүн $i+m$ -р нүгнээ абаарат бол

$$S_{k+1} = S_k + (f(x_{i+m}) - f(x_{i+m-1}))g(i+m)$$

да $m \leq x_i$ дагна (өгөөн нөхүүлээр). Төхирөмжтэй f, g -г сонгон, $S_{k+1} - S_k$ -г зааглаар, үнэлгээ гаргаар.